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Å-SCALE DISPLACEMENTS REVEALED BY X-RAY MOIRÉ TOPOGRAPHS

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Moiré fringes in electron microscope images of overlapping simultaneously reflecting crystals have been much studied since it was found that they could reveal dislocations[2,3] and stacking faults[3] in either crystal, or the misfit between an epitaxial layer and its substrate.[3] In the case of x rays, the fringes observed in the x-ray interferometer[4] are essentially moiré fringes produced by simultaneously reflecting crystals which are spatially separated. Moiré fringes also appear in x-ray topographs where a crack divides a crystal into two overlapping, simultaneously reflecting layers. The importance of x-ray moiré fringes arises from the much greater fringe spacing observable on x-ray topographs compared with electron micrographs: the ratio is typically 10^4. Hence x-ray moiré topographs can detect relative rotations and relative differences in interplanar spacing with 10^4 times the sensitivity possible in the electron case. It should be possible, moreover, to measure these quantities simultaneously over an area of overlap equal to the area of the whole topograph image, about 5 cm^2. This would permit the direct comparison of interplanar spacings of two perfect (or nearly perfect) crystals to one part in 10^7 to 10^8,

The basic geometrical interpretation of moiré patterns is the same for electron, x-ray, and light optics. Let the radiation pass successively through the two periodic media A and B (crystals in the x-ray and electron case) and be diffracted by planes of reciprocal vector $\mathbf{g}_A$, spacing $d_A = 1/|\mathbf{g}_A|$ in A, and correspondingly in B. Then the reciprocal vector of the moiré fringe system is $\mathbf{G} = \mathbf{g}_A - \mathbf{g}_B$, and the moiré fringe spacing D is $1/|\mathbf{G}|$. Conveniently one may consider two special cases. The pure "rotation" moiré pattern, in which $d_A = d_B = d$, but $\mathbf{g}_A$ makes a small angle ϵ with $\mathbf{g}_B$, has fringes of spacing $D = d/\epsilon$, with $\mathbf{G}$ perpendicular to $\mathbf{g}_A$ (or $\mathbf{g}_B$). Secondly, the pure "compression" moiré, in which g_A and g_B are parallel but $d_A \neq d_B$, has fringes of spacing $D = d_A d_B / (d_A - d_B)$, with **G** parallel to g_A. Both special cases appear in parts of the fringe patterns shown in Figs. 1(a)–(c). These are x-ray projection topographs[5] of a crack in a quartz plate, 0.45 mm thick, which has propagated inwards from the edge of the plate (just below the bottom of the field of view). A region of severe distortion causes a strong enhancement of diffracted intensity from central parts of the crack, but nearer its intersections with the plate surfaces moiré fringes are clearly seen, especially in the upper part of the crack. The horizontal fringes on the right-hand regions of the crack in Figs. 1(a) and (b), which are close to the crack intersection with the plate surface facing the x-ray source, correspond to a rotation moiré of angle $\epsilon = 2''$, since in Fig. 1(a) $d = 4.25$ Å and $D = 44$ μ; and in Fig. 1(b) $d = 2.13$ Å and $D = 22$ μ. At the top left of the crack in Fig. 1(c) a few fringes run vertically with spacing 20 μ. These correspond to a compression moiré

(a)

(b)

(c)

Fig. 1. Projection topograph of moiré fringes at a crack which runs nearly normal to a quartz plate oriented a few degrees off (10$\bar{1}$1). Direction of view is along the diffracted beam, towards the plate, and makes about 45° with the plane of the crack since the Bragg planes are inclined about 45° to the plate and the Bragg angle is small. Far edge of crack is on right, near edge on left. Field width 0.7 mm. Projection of diffraction vector is horizontal. Ag $K\alpha$ radiation. (a) 10$\bar{1}$0 reflection; (b) 20$\bar{2}$0 reflection; (c) repeat of 20$\bar{2}$0 reflection after partial relaxation.

with $d_A - d_B = 2 \times 10^{-5}$ Å. The topograph Fig. 1(c) was taken after an interval during which the crack was examined optically and immersion oil was allowed to run into it. It was found subsequently that a considerable relaxation of the displacements across the crack had taken place, as can be seen by comparing Figs. 1(c) and (b). It is clear that repetitive topography can reveal fringe movements of $^1/_5$ D or less on widely spaced fringes. (This measurement is facilitated by use of "landmarks" such as the images of strain-fields of inclusions which appear as the black "double-dots" scattered over the field of Fig. 1.) A fringe shift of $^1/_5$ D corresponds to a displacement of crystal A relative to crystal B of only 0.4 Å, in the case of the reflection of Figs. 1(b) and (c).

The fringes at this and other cracks exhibit the following characteristics, which are in accord both with the simple geometrical analysis given above and with expectations from the dynamical electron diffraction theory of moiré fringes[6,7] appropriately applied to the x-ray case.

1. The moiré fringe pattern in the reflection *nh*, *nk*, *nl* is geometrically similar to that in the reflection *hkl*, but $D(nh, nk, nl) = (1/n)/D(hkl)$. This is the unfailing diagnostic characteristic identifying moiré fringes.

2. Moiré fringe contrast is strongest when the component of **G** normal to the Ewald sphere is smallest. The fringes are then modulated with a depth periodicity of half the Pendellösung period[6,7] [this is seen well in the upper right-hand fringes in Fig. 1(a)]. When the component of **G** normal to the Ewald sphere is relatively large, so that coherent simultaneous reflection by crystals A and B no longer occurs, the moiré fringes disappear and only depth contours with the full Pendellösung periodicity appear, as they do in images of tapering crystals[8] and of low-angle boundaries.[9] Usually a complicated pattern of crossed moiré and depth contours appears.

3. The moiré pattern in general differs in reflections from different Bragg planes, because each reflection is sensitive to different components of the relative "rotation" and "compression" of the two crystals.

The clear visibility of moiré fringes formed by two parts of a crystal separated by a crack indicates the feasibility of observing moiré fringes over a large area where two perfect crystals are super-imposed. Moreover, if one crystal were attached to the moving member of an optical interferometer then the number of interplanar spacings contained in a standard light wavelength could be found by counting simultaneously the moving x-ray moiré and light interference fringes.

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