

*Acta Cryst.* (1968). A 24, 246

**Use of the Ewald sphere in aligning crystal pairs to produce X-ray moiré fringes.** By J. BRÁDLER\* AND A. R. LANG,  
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(Received 17 July 1967)

One crystal plate can be superposed upon another of the same material to produce observable X-ray moiré fringes in the simultaneously diffracted beam transmitted through the plates. During alignment it is necessary to determine, and then minimize, the small difference vector between the acting reciprocal lattice vectors. By rigid rotation of the crystal pair about the mean reciprocal lattice vector the Ewald sphere is caused to make various angles with the difference vector. This vector can then be fully determined by measuring its component normal to the Ewald sphere at two or more settings of the sphere.

We have recently developed methods for superposing one crystal plate upon another so as to produce X-ray moiré fringes with repeat periods in the range 10–500 microns, such fringes being easily photographed by standard X-ray topographic techniques. Previous X-ray moiré fringe work has been done with (a) X-ray interferometers (Bonse & Hart 1965*a, b, c*; 1966*a, b*; 1968) in which parts of a single crystal of silicon are milled away to leave just the interferometer elements upstanding; (b) superposed nearly parallel growths of platelet cadmium sulphide crystals (Chikawa, 1965); and (c) cracks in quartz at which moiré fringes occur owing to the relative displacement of the two sides of the crack (Lang & Miuscov, 1965). In all the above-cited works the fringes have been formed between elements of crystal grown in a perfect or nearly perfect alignment which has been little disturbed since growth. To produce moiré fringes between crystal elements which have been physically separated from each other involves the stringent requirement of bringing their respective Bragg-plane normals together within a solid-angle range as small as  $10^{-10}$  steradian. This process of alignment, apparently arduous, has been turned into a routine operation by a simple exploitation of the well-known Ewald sphere geometry.

The experimental problem is illustrated by Fig. 1 which shows the region of reciprocal space containing  $A$ , the tip of the operating reciprocal lattice vector,  $g_A$ , of one crystal slice, and  $B$ , the tip of  $g_B$ , the operating reciprocal lattice vector of the second slice. The vector  $g_A$  points directly towards the observer, and  $g_B$  nearly so. The crystals are of the same species, with closely similar lattice parameters, and use the same Bragg reflexion, so that  $|g_A| = |g_B|$  very nearly. The crystal plates themselves are each used in symmetrical transmission and one is placed closely behind the other so that both incident and diffracted beams leaving the first crystal subsequently pass through the second crystal. In Fig. 1  $X'AX$  is the trace of the plane containing the rays incident upon and diffracted by a given element of the first crystal. Since the slit-collimated incident beam has a divergence in this plane at least ten times the angular range of reflexion of the crystal, both the first and second crystals are readily set by coarse angular adjustments to realize their

full integrated reflexions. The trace of the Ewald sphere for the ray in the plane  $X'A$  which satisfies the Bragg condition exactly in the element of the first crystal considered is shown by  $YAY'$ , and is sufficiently closely represented by a straight line in the small region of reciprocal space involved. In order to produce observable moiré fringes two conditions must be satisfied. Firstly,  $B$  must be brought to lie within a thin band centred on  $YY'$  so that the angular range of rays which undergo Bragg reflexion at the second crystal falls within that of rays Bragg reflected by the first crystal. For practical purposes, with the crystals and radiations used, this range can be taken as  $\pm 1''$  of arc. It is indicated by the vertical dashed lines on Fig. 1. The distance of  $B$  from the Ewald sphere is called  $\gamma$ . The second condition is that the distance of  $B$  from  $X'X$  be not too large for moiré fringes to be resolved. The moiré fringe period (when  $|g_A| = |g_B|$ ) is inversely proportional to this distance which we call  $\tau$ . For easy recognition on fast coarse-grained X-ray film a fringe spacing not finer than 40 microns is desirable and this corresponds to a range of  $\pm 1''$  of arc with the silicon 220 reflexion, and is indicated by the horizontal dashed lines.  $B$  is 'home' when it lands in the obliquely shaded area of small  $\gamma$  and small  $\tau$  surrounding  $A$ . Now  $\gamma$  can be determined, and hence minimized, with progressive fineness, by observations in succession on the half width of the overlapping reflexions from the two crystals, the peak reflected intensity, and the intensity profile across the diffracted beam. On the other hand there is no simple method for finding  $\tau$  when it is fairly small (say less than  $100''$ ) but still outside the shaded region. The problem is solved by rigid rotation of the crystal pair about the mean reciprocal lattice vector, so that, in a coordinate system fixed to the crystals, as used in Fig. 1, the trace of the Ewald sphere is tilted away from the vertical. A rotation of the crystals clockwise by  $45^\circ$  (looking towards the reciprocal lattice origin) brings the Ewald sphere trace to  $E_1E_1'$ , and an anticlockwise rotation by  $45^\circ$  from the initial position brings it to  $E_2E_2'$ . If in each position  $\gamma$  (defined as the distance from the Ewald sphere in its current position) is made very small by appropriate movement of  $B$  normal to the Ewald sphere, then  $B$  is 'home' in just two, independent, steps: *viz.* the step from  $B$  to  $B'$  and from  $B'$  to  $A$ . In practice a few, but not many, more steps may be needed. On the other hand it is found that sufficient

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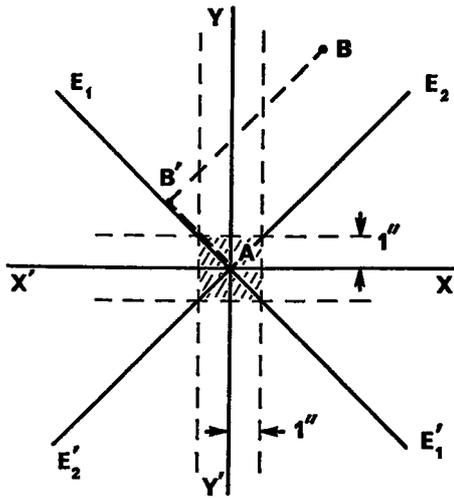


Fig. 1. Region of reciprocal space including the tips *A* and *B* of the active reciprocal lattice vectors of the first and second crystals.

independence of the steps is obtained with rotation ranges as little as half the  $\pm 45^\circ$  shown in Fig. 1.

Fig. 2 shows an example of the beautifully regular fringes formed by superposing one plate of perfect silicon upon another. The plates were about one millimetre thick, so that for each  $\mu t \sim 1$ . Equally clear and straight fringes were observed over an area of  $45 \text{ mm}^2$ . The fringes represent a pure rotation moiré, the rotation, as determined from the fringe period of 17 microns, is  $2\frac{1}{2}''$  of arc.

#### References

- BONSE, U. & HART, M. (1965a). *Appl. Phys. Lett.* **6**, 155.  
 BONSE, U. & HART, M. (1965b). *Appl. Phys. Lett.* **7**, 99.  
 BONSE, U. & HART, M. (1965c). *Z. Physik*, **188**, 154.  
 BONSE, U. & HART, M. (1966a). *Z. Physik*, **190**, 455.  
 BONSE, U. & HART, M. (1966b). *Z. Physik* **194**, 1.  
 BONSE, U. & HART, M. (1968) *Acta Cryst.* **A24**, 240.  
 CHIKAWA, J.-I. (1965). *Appl. Phys. Lett.* **7**, 193.  
 LANG, A. R. & MIUSCOV, V. F. (1965). *Appl. Phys. Lett.* **7**, 214.

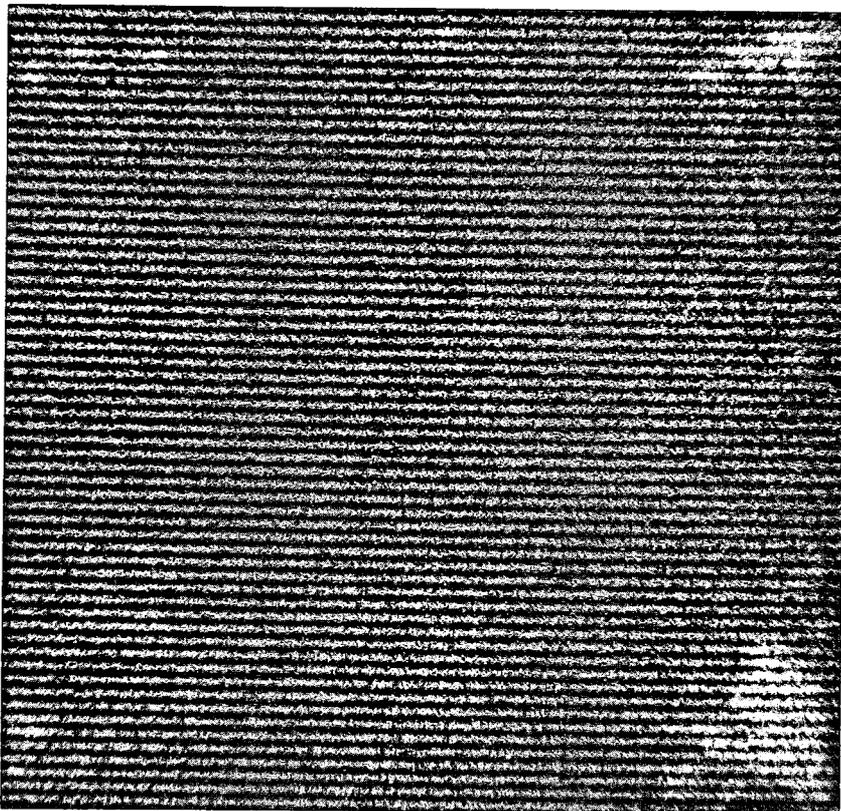


Fig. 2. Pure rotation moiré formed by superposition of one silicon plate upon the other. Reflexion 220, reciprocal lattice vector horizontal, radiation  $\text{Mo K}\alpha$ . Field 1 mm square. Recorded on Ilford L4 nuclear emulsion.