On magnetic domain configurations in single-crystal, (112)-orientation plates of iron-silicon alloy: some X-ray topographic observations and their interpretation

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Possible magnetization states of a cubic ferromagnetic crystal having easy magnetization directions parallel to the cube axes (as in Fe+3 mass % Si alloy) can be represented using a regular octahedron, as first shown by Néel. This model is developed to derive rules that govern shapes and relative volumes of the differently magnetized domains needed to produce mean magnetization lying in a plane of any orientation. Single-crystal plates of Fe+3 mass % Si alloy not containing an easy-magnetization direction, such as plates parallel to (112) studied in this work, are in general subdivided into a hierarchy of domains that in spatial scale diminish from specimen interior towards specimen surfaces. X-ray topography offers a non-contacting method of mapping domain configurations. Under appropriate diffraction conditions the strains associated with $g_00_0$ Bloch walls generate strong diffraction contrast. In X-ray transmission topographs the superimposition of contrast from surface closure domains upon that of inner domains generally gives rise to highly complex images from (112) plates. However, with specimen thicknesses less than 20 $\mu$m the patterns can become sufficiently simplified to make possible the identification of shapes and magnetization axes of all domains present. A pattern in this class is illustrated, and interpreted with the aid of the octahedron. Reasonable agreement is found between the observed size scale of the domains and that calculated for minimum energy in the domain structure proposed.

1. Introduction

The directions of easy magnetization in crystals of Fe+3 mass % Si alloy are the crystallographic cube axes. In the case of crystal plates not containing a cube axis, and in absence of applied field, the need to avoid high magnetostatic energy associated with surface magnetic charge causes the bulk of the plate to be divided into a lamination of domains with different magnetization directions combined in such proportions that the mean magnetization vector has no component normal to the plate. In absence of any easy magnetization direction parallel to the plate surface, reduction of magnetostatic energy has to proceed by diminishing the size...
scale of areas of alternating positive and negative polarity on the surface. Therefore, close to the plate surface the inner domains subdivide into smaller domains, and further subdivision will form a hierarchy of domains until, broadly stated, decrease in magnetostatic energy is balanced by the energy increase due to increase in domain wall area. Bitter patterns on surfaces parallel to (112) confirm presence of this hierarchy, at least in the plate thicknesses of ca. 20 μm and above, which are sufficiently robust to be studied by this technique.

The earliest X-ray topographic studies of internal magnetic domain structures in iron–silicon alloy single crystals were carried out on plates in either (110) or (112) orientations, the diffraction contrast being generated by strains associated with 90° domain wall boundaries. The high density of such boundaries in (112)-orientation plates provided the patterns rich in detail that were chosen to illustrate the ability of this non-contacting technique to image domains and record their movements (Polcarová & Lang 1962). In the areas concerned the Bitter patterns showed domain subdivision down to a scale of about 1 μm, whereas the X-ray topographic images contained contrast components corresponding to the interior laminated structures and had periods in the 10 to 15 μm range. Most published X-ray topographic studies of magnetic domains in Fe–Si have been performed on specimen plates parallel to at least one direction of easy magnetization; and in (001) plates a variety of phenomena have been investigated, e.g. the zig-zag fine structure of 90° domain walls (Polcarová & Kaczér 1967; Polcarová & Lang 1971) and the strainfields of ‘tree’ closure domains (Miltat 1976). However, on specimens containing no direction of easy magnetization there is an important work combining Bitter pattern, Kerr effect and X-ray topographic observations on (111)-orientation plates (Labrune & Kléman 1973).

Quite long ago Néel (1944) made use of a regular octahedron to represent the possible states of magnetization of a cubic ferromagnetic crystal having easy magnetization directions parallel to (100). Here this model is developed to illustrate the rules governing the relative volumes of differently magnetized domains required to produce mean magnetization in a plane of any orientation, and those governing the permissible orientations of walls between the component domains such that formation of free poles within the specimen is avoided. In §3, observations on a type of domain pattern that occurs in (112)-orientation plates when the specimen thickness is less than 20 μm are described, and in §4 the usefulness of the octahedron in interpreting such patterns is demonstrated. To account for the scale of the pattern requires considering likely closure domain structures, and this problem is discussed in the concluding sections.

2. The magnetization octahedron

It is convenient to label the vertices of the octahedron as shown in figure 1 so that the six vectors OA, OĀ, OB, etc., are in the directions [100], [100], [010], etc. The lengths OA, OB, etc. correspond to the saturation magnetization $I_s$. Then, as Néel pointed out, the mean magnetization vector of a specimen with zero internal field, having various proportions of its volume in domains magnetized along each of the six easy directions, is represented by the vector from O to any point in the octahedron; while points between this octahedron and the circumscribing sphere of radius OA (corresponding to $I_s$) become accessible under applied field.

Here more extensive use of this octahedron is made, noting the following points.
1. Single domain situations correspond to the vertices of the octahedron.

2. Combinations of just two kinds of domain correspond to points in the lines joining vertices: (a) 90° two-component configurations correspond to points in the edges of the octahedron, e.g., a combination of A and B domains to a point in the line AB; (b) 180° two-component configurations correspond to points in the diagonals of the octahedron, e.g., a combination of A and A to a point in the line AA; (c) the volume proportions of the two components are in such cases as the distances from the opposed vertices, e.g., for the representative point s in AB, the volume fraction of A domains is measured by sB, the volume of B domains by sA.

3. Combinations of just three kinds of domains correspond to points within triangles whose vertices are the corresponding vertices of the octahedron: (a) three-component 90° configurations correspond to points in the faces of the octahedron, e.g., a combination of A, B, and C domains to a point in the face ABC; (b) three component 90° and 180° configurations correspond to points in diagonal planes (webs) of the octahedron, e.g., a combination of A, A and B gives a point within the triangle AAB; (c) in these cases, the volume proportions of the three kinds of domain are as the distances of the representative points from the opposite sides of the triangles, divided by its corresponding height, e.g., for a point s in ABC, the proportions of A, B and C domains are as the distances of s from BC, CA and AB respectively, and for a point s in AAB the proportions of A, A and B domains are as the distances of s from AB, s from AB and \(\sqrt{2}\) times the distance of s from AA.

4. Any further combination can be displayed according to the same principles: thus if any configuration with representative point \(s_1\) is combined with another representative point \(s_2\), the resultant representative point \(s_3\) lies in the line \(s_1s_2\), and rule 2(c) governs the volume proportions of configurations 1 and 2 in the more complex configuration 3.

5. If domains with magnetization vectors \(I_1, I_2\) meet in the plane whose unit normal vector is \(n\) and div \(H\) is zero, magnetic continuity (div \(B = 0\)) requires

\[ I_1 \cdot n = I_2 \cdot n, \quad \text{that is} \quad (I_2 - I_1) \cdot n = 0.\]

This leads to a simple geometrical rule, describable in terms of the magnetization

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Figure 1. Octahedron showing vertices pointing in directions [100], [100], [010], etc., labelled A, A, B, etc., respectively. Direction of view perpendicular to edge [011] = BC, and making about 6° with the (112) plane drawn containing the octahedron centre, O.
octahedron, for determining possible meeting surfaces between domains, i.e. surfaces with no divergence of $H$, conveniently called 'pole-free'. The necessary and sufficient condition that the meeting surface between domains with representative points $s_1$ and $s_2$ is pole-free is that it contains the straight line $s_1s_2$; it can be any plane, prism, or cylinder, containing this line as a generator. Thus, for example, domains $A$ and $B$ can meet in a pole-free manner on any surface containing the line $AB$, that is, belonging to the zone $[\bar{1}00]$, or domains $A$ and $A'$ meet pole-free on any surface containing $AA'$, that is, any surface belonging to the [100] zone. (In practice this equivalence of domain wall orientations will be lost when the dependence of domain wall energy and magnetoelastic energy upon wall orientation is taken into account.)

6. This rule applies equally to the meeting of compound domains one with another, save that in such a case the vector $O_s$ represents the mean magnetization of the compound domain, and the rule governs the mean meeting surface: it may have a fine structure of faceting governed by the same rule applied to its component parts.

7. The two simplest structures for a two-component compound domain are respectively planar laminations and rods of one kind of domain in a matrix of the other. Rods must have the direction $s_1s_2$. Laminations must be on planes of this zone. Subject to the zone rule, bending of the laminations is permissible, even into structures of maze-like cross-section which may not be readily classified either as laminations or rods. The zone-rule permits edge dislocations of the laminated magnetic structures, but forbids screw dislocations of them (thus depriving them of one mechanism which would have provided for the multiplication of laminations to their equilibrium spacing).

8. A three-component parallel lamination must satisfy two zone rules simultaneously: thus the plane of laminations is uniquely defined, e.g. a three-component lamination of $A$, $B$ and $C$ components must be laminated parallel to the triangle $ABC$, that is, the plane $(111)$. A three-component lamination of $A$, $A'$ and $B$ must be parallel to a plane containing the directions $AB$ and $\bar{A}B$, i.e. the plane $(001)$.

9. In the absence of an external field the normal component of magnetization at a free surface must be zero. Permissible magnetization vectors are thus found by cutting a section through the centre of the octahedron with a plane parallel to the free surface in question. In a general orientation this plane contains no directions of easy magnetization, and is a hexagon. Its vertices represent six alternative 90° two-component systems of compound domains giving satisfaction of the surface condition in the mean. Its centre corresponds to three alternative 180° two-component systems satisfying the surface condition in the mean. All other points in the hexagon correspond to systems of more than two components.

10. In the general orientation, the pole-free surface condition can never be satisfied except in the mean. Any single domain reaching the external surface gives rise to external field, and a corresponding excess of energy, which can be reduced by the interposition of surface closure domains. However, the presence of surface closure domains does not prevent the application of the geometric rules of 9 to determine permissible mean magnetization vectors of the underlying structure, provided that the surface closures make a recurrent structure.
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Figure 2. Stereographic projection on (112) showing traces of cube planes. The direction [111] makes 39.2° with [201] and [021].

Figure 3. Magnetic domain patterns typical of a (112)-orientation plate specimen where the thickness exceeds ca. 20 μm. Direction [110] points horizontally to left. Field width 0.5 mm. (a) X-ray transmission topograph, CoKα, radiation, 110 reflexion. (b) Bitter pattern of same area.

3. Observations

The preparation of thin single-crystal plates of iron-silicon alloy, and the X-ray topographic techniques used to examine them, were described by Lang & Polcarová (1965). That work dealt with (110)-orientation plates. The specimen of present concern was the thinnest of several parallel to (112) examined. It was blade-like in shape, and the sub-grain at the blade tip, length about 6 mm and maximum width 3.5 mm, contained the patterns of particular interest. One edge of the blade had been...
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reduced to a feather edge in the process of electrolytic polishing. As explained by Lang & Polecarová (1965) the specimen thickness can be determined without touching the specimen from the local Pendellösung fringe order on X-ray topographic images. Combining fringe counts using different X-ray wavelengths and Bragg reflexions enables the local thickness to be determined with an uncertainty of only about 1 μm in the thickness range from about 3 to 50 μm in Fe–Si crystals. In the thin (112)-orientation specimen, diffraction contrast from the domain structure became clearly recognizable when the thickness reached about 8 μm. From that thickness up to about 20 μm the domain structures appeared as simple stripe patterns on the topographs, with no modulation of visibility along the stripes. At greater thicknesses the stripe pattern persisted, with stripe periods increasing with specimen thickness, but it became less distinct due to addition of two-dimensional mottle or trellis-like patterns that were attributed to the strainfields of surface closure domains. Such was concluded from examining images of thin regions taken with the 110 reflexion recorded with AgKα, MoKα, and CoKα radiations, reflexion 011 with AgKα and MoKα, and reflexions 101 and 101 with CoKα. (See stereographic projection on (112), figure 2.)

Figure 3 compares X-ray and Bitter patterns of an area where although the thickness only slightly exceeds 20 μm both images exhibit the general characteristics of patterns seen in all thicker regions of (112) plates examined. An inclusion in the crystal (lower right of centre in the photographs) provided a landmark for matching the image areas. On the X-ray topograph the strainfield of the inclusion produces intense diffraction contrast, seen as blackening on the image; and some areas are rotated completely away from Bragg-reflecting orientation (though with the highly collimated incident beam used, a milliradian rotation about the goniometer axis is sufficient to achieve this). Repeated X-ray topographs of Fe–Si plates reveal apparently spontaneous changes of domain patterns, so exact correspondence cannot be expected between the Bitter pattern recorded in Prague and the X-ray topograph recorded in Bristol, but the inclusion exercises a local stabilising effect on the domain configuration. (However, after application of an external magnetic field substantial pattern changes occurred (Polecarová & Lang 1962).) Except in the uppermost parts of the field, the stripe patterns seen in the X-ray topograph are parallel to [021] and [021]. To recognize the stripes in the mottled pattern left of the inclusion it helps to sight the pattern along [021] at a low grazing angle with the photograph.

Figure 4a–c shows the simpler class of pattern, hereafter referred to as the ‘geometric pattern’, which is observed within the specimen area that lies between the 20 μm thickness contour and the minimum thickness for domain contrast to appear. These micrographs exhibit its clearest example, which was found in an area free from low angle boundaries and with a low density of randomly distributed crystal lattice dislocations. However, in this vicinity the specimen had a slight bend (radius between 1.5 and 2 m), which reduced the field that could be satisfactorily imaged at one angular setting of the crystal in the case of 011 Bragg reflexions. This circumstance determined the choice of orientation of the fields reproduced in the

Figure 4. X-ray topographs of the ‘geometric’ domain pattern observed where specimen thickness is less than 20 μm. Direction [201] points to left, rotated clockwise a few degrees from the horizontal. Field width 0.65 mm. View looking towards X-ray source. In all cases the angle between diffracted beam and [112] is small enough for pattern distortion due to obliquity of view to be negligible. (a) AgKα, radiation, reflexion 011. (b) MoKα, radiation, reflexion 011. (c) CoKα, radiation, reflexion 101.

figures. The specimen thickness at the field centre is 16 μm. It tapers to 13 μm in the top right corner, i.e. towards [111]. Features to note include the following. The area consists principally of two fields of stripes whose contrast has no modulation parallel to their length. The stripes lie roughly parallel to [201] in the upper and right-hand field, and roughly parallel to [021] in the lower left field. The well-defined field boundary traces are accurately [111] (lower left to upper right) and [110] (upper left to lower right), and are lines of symmetry of stripe directions in the pattern. Some domain movements occurred between recording figure 4a and at a later date figure 4b, c: note in particular the disappearance of a small square island of [021] stripes from the field of [201] stripes close to the bottom edge of figure 4a, and in figure 4b, c the widening of the rectangular promontory of [021] stripes above and left of centre. The period of [201] stripes is 12.5 μm in the figure centre, and diminishes to about 8 μm in the top right of figure 4b where several edge dislocations of similar sign in the stripe pattern can be seen. The period of [021] stripes where they are most distinct, left of centre, is about 14 μm. The low contrast of the stripe pattern overall can be judged qualitatively by comparison with the much denser blackness of images of dislocations that thread the specimen between its near and far surfaces. Their images are the variously oriented, generally short black lines randomly distributed over the figures. Those lines appearing in all images have Burgers vector parallel to [111], those seen in figure 4a, b only have Burgers vector parallel to [111] and those seen only in figure 4c have Burgers vector parallel to [111]. Weaker contrast is seen in those stripes making the smaller angle with the diffraction vector; this is especially notable in the case of [201] stripes in the [101] reflexion, figure 4c.

### 4. Interpretation

The stripe pattern in figure 4 suggests lamination of the whole area on planes approximately perpendicular to the plate, the lamination period being the repeat period of the stripe period. An interpretation of the pattern must provide a satisfactory explanation of the rectilinear boundary, traces parallel to [111] and [110], that separates the [201]-stripe field from the [021]-stripe field. If the lamination is accomplished using only a two-component system of magnetization directions in each field (apart from closure domains) then four possibilities arise for the pair of fields that give the necessary symmetry about [111] and [110], two with 180° lamination walls, and two with 90° walls. Recall from the penultimate paragraph of §2 that allowable 90° two-component systems produce net magnetization in the plane of the plate, whereas 180° two-component systems do not. Adopting the notation for magnetization directions used in figure 1, the two possibilities for 180° lamination walls are (i) magnetization directions C and Ĉ in both fields, and (ii) magnetization directions A and À in one field, and B and B in the other field.

In system (i) the lamination walls can take any orientation in the [001] zone, but (100) and (010) would have minimum energy density (see, for example, Craik & Tebble 1965, table 2.3). Thus lamination walls in one field would be fairly close to (100) and in the other field to (010). Segments of field boundaries across which no change of magnetization occurs will be termed null-wall segments. In this system the entire field boundary is of null-wall type, and not in itself a generator of diffraction contrast.

In system (ii) minimum wall energy densities would be achieved with lamination
walls (010) in the AA field (AA symbolizing lamination consisting of equal parts of A and A domains), and (100) in the BB field. Using the symbol A/B to denote a wall between A and B domains, the magnetization octahedron shows that a field boundary made up of strips of A/B, A/B, A/B... must contain [110], and one containing the sequence A/B, A/B, A/B... must contain [110]. The field boundaries could lie in (110) and (111), respectively, both normal to the plate.

Turning to possibilities with 90° lamination walls, it is necessary to consider flux matching at field boundary walls in addition to the condition that net magnetization in each field is parallel to (112). From figures 1 and 2 it is seen that there are six possible net magnetization directions, which are [110], [021], [201] and their inverses. The corresponding two-component systems are A/B, B_{1/2}/C, A_{1/2}/C and their inverses (B_{1/2}/C symbolizing a lamination system composed of B lamellae twice as thick as C lamellae). Excluding null-wall character for the entire field boundary, and hence excluding the AB and AB lamination systems, leaves the systems B_{1/2}/C, A_{1/2}/C and their inverses for consideration regarding flux matching at the field boundaries. The possible gross matchings at the (110)-trace field boundary are A_{1/2}/C with B_{1/2}/C, and between their inverses. The possible gross matching at the (111)-trace field boundary are A_{1/2}/C with B_{1/2}/C, and between their inverses. Thus there are no gross matchings common to both field boundary orientations. Hence one or other of the fields must contain 180° walls, which meet the field boundary at junctions of (110)-trace and (111)-trace segments. If the configuration of the field boundary is such that in one field no lines parallel to the net magnetization cross both a (110)-trace segment and a (111)-trace segment then the 180° walls can lie in that field alone. In general, there will be 180° walls in both fields.

Interpretation of figure 4 will now be pursued in detail, on the basis that it consists of 90° lamination systems containing 180° walls where necessary. Consider the system A_{1/2}/C (and its inverse). Its 180° walls could be between A and A, lying in the zone of [100], or between C and C, lying in the zone of [001]. The lamination wall is in zone [101]. The plane common to both zones [100] and [101], and to zones [001] and [101], is (010). Thus 180° walls in the system A_{1/2}/C, and their closely adjacent lamination walls, should lie in (010) and produce a trace parallel to [201]. Accordingly, the upper right field of figure 4 is identified with the A_{1/2}/C (and A_{1/2}/C) system, and the lower left field with the B_{1/2}/C (and B_{1/2}/C) system. Where 180° walls are needed in the upper field, thicker stripes are seen in figure 4a, b. Figure 5 gives an interpretation of the principal features associated with the field boundary that appear in figure 4, incorporating the gross and detailed field boundary matching arrangements selected. (This configuration with all directions reversed is of course equally probable.) Since the stripes associated with 180° walls in the upper field are exceptionally wide it has been assumed that all these walls lie between A and A. The single 180° wall needed in the lower field is imaged only indistinctly but does not give rise to an exceptionally wide stripe, and it is assumed to lie between C and C. Figure 4a contains some indications that detailed arrangements at corners may be somewhat different from those drawn in figure 5. Regarding the detailed geometry of the field boundary walls, the arrangements selected, in which strips of equal width meet (e.g. narrow C does not meet wide A or B) is clearly to be preferred. Moreover it offers simpler geometry for the constituent facets of both [111]-direction and [110]-direction segments of the field boundary. Specifically, in the case of the former, the A/B facet lies in the [110] zone and the C/C facet in the [001] zone. The plane common to these two zones is (110) and so a non zig-zag wall parallel to (110) is
walls (010) in the AA field (AA symbolizing lamination consisting of equal parts of A and Â© domains), and (100) in the BB field. Using the symbol A/B to denote a wall between A and B domains, the magnetization octahedron shows that a field boundary made up of strips of A/B, A/B, A/B ... must contain [110], and one containing the sequence A/B, A/B, A/B ... must contain [110]. The field boundaries could lie in (110) and (111), respectively, both normal to the plate.

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domains; their cross-section profiles drawn assume that they have uniform magnetization \( I_\perp \) lying parallel to the specimen surfaces, and, specifically, parallel to \( \pm [152] \). The energy of this structure will be derived below, but first, as an important yardstick for judging the energy-reducing effect of closures, the energy of the two-component lamination structure without any closure domains will be calculated. The geometry of this simpler structure is derived from that of figure 6 by extending the 90° lamination walls parallel to (010) to meet the top and bottom specimen surfaces. The energy calculation follows established procedure (Kittel 1946). The magnetic pole density \( \rho(x) \) is equal to the component of magnetization normal to the surface; \( \rho_1(x) \) and \( \rho_2(x) \) denote its values on the surfaces of the thinner, (1), and thicker, (2), domains respectively. On the upper surface, in the left part of figure 6, \( \rho_1(x) = (2/\sqrt{6}) I_\perp \) and \( \rho_2(x) = -(1/\sqrt{6}) I_\perp \). With \( \rho(x) \) expressed by the Fourier series

\[
\rho(x) = \sum_{n=-\infty}^{\infty} a_n \cos(2n\pi x/3D'),
\]

in which \( 3D' = 3D \sec \beta \), the lamination period on the specimen surface, the Fourier coefficients are \( a_n = [(\sqrt{6}) I_\perp /n\pi] \sin(\frac{1}{2}n\pi) \). When \( T \) is sufficiently greater than \( D \) for interaction between charges on top and bottom surfaces to be neglected, a condition satisfied when \( \exp(-2\pi T/3D) \ll 1 \), the magnetostatic energy per unit area of specimen, \( E_D \), including both top and bottom surfaces (see, for example, Craik 1971) is

\[
E_D = (3D')^2 \sum_{n=-\infty}^{\infty} a_n^2 /n. \tag{1}
\]

Inserting the values found for \( a_n \) into (1) gives \( E_D = (27D'T_\perp^2 /4\pi^2) \sum_{n=1}^{\infty} n^{-3} \), with \( n = 1, 2, 4, 5, 7, 8 \), etc. Now \( \sum_{n=1}^{\infty} n^{-3} \) with \( n \neq 3m \) is (26/27) \( \sum_{k=1}^{\infty} k^{-3} = (26/27) 1.202 \). This summation yields

\[
E_D = 0.79D'T_\perp^2. \tag{2}
\]

Reference to figure 6 shows that the domain wall energy per unit area of specimen,

$E_w$, amounts to $E_w = 2\gamma T \sec \beta / 3D'$, $\gamma$ being the energy per unit area of a cube-plane 90° wall. Minimizing the total energy $E = E_D + E_w$ with respect to $D'$ to find the equilibrium value $D_0'$ gives

$$D_0' = (0.84\gamma T \sec \beta / I_3)^{1/3}. \quad (3)$$

Putting in the appropriate values $I_3^2 = 2.61 \times 10^6 \text{J m}^{-2}$, $\gamma = 0.8 \text{mJ m}^{-2}$ (Kaczér 1957), and $T = 16 \mu\text{m}$ in the centre of the area under study yields $3D_0' = 0.69 \mu\text{m}$, about 20 times smaller than the observed stripe period. Even with this small value of $D_0'$, the magnetostatic energy has the high value $E_D = 48 \text{mJ m}^{-2}$. The magnetostrictive strain energy of the two-component lamination can be calculated. Assuming $D < T$, it is $\frac{1}{3}Y\lambda_{100}^2 / (1 + \nu)$ per unit volume. Taking Young's modulus $Y = 203 \text{GPa}$, Poisson's ratio $\nu = 0.29$ and magnetostriction $\lambda_{100} = 2.7 \times 10^{-5}$ gives $38 \text{J m}^{-2}$ for the elastic energy density, which with $T = 16 \mu\text{m}$ corresponds to magnetostrictive energy per unit area of specimen $E_\lambda = 0.61 \text{mJ m}^{-2}$.

Turning to the structure with closure domains drawn in figure 6, a convenient expression for the anisotropy energy $E_K$ in closure domains magnetized in the plane (112) is

$$E_K = (K_1/96)(21 + 4 \cos 2\varphi + 7 \cos 4\varphi), \quad \text{(4)}$$

where the angle $\varphi$ is measured from the [111] direction, $K_1$ is the first anisotropy constant and the contribution from $K_2$ is neglected (Bozorth 1951). This energy is least for $\cos 2\varphi = -1/7$, corresponding to $\varphi = 49.1^\circ$, and for this angle $E_K = K_1/7$. Also, $\varphi$ is then close to the angle between [111] and [152], 50.8°. Hence magnetization in the closure domains will lie practically in the plane of the figure, and in the following calculation is assumed to lie in that plane exactly. In lamination domains of both types (1) and (2) let $I_\perp$ be resolved into components $T_1$ and $T_2$ in the plane of the figure and parallel to the lamination walls, and $P_1$ and $P_2$ perpendicular to the plane of the figure. Magnitudes of these components are, in (1), $T_1 = (2/\sqrt{5})I_\perp$ and $P_1 = (1/\sqrt{5})I_\perp$; and in (2), $T_2 = (1/\sqrt{5})I_\perp$ and $P_2 = (2/\sqrt{5})I_\perp$. The depth, $h$, of the closure domains is determined by flux closure considerations: $hI_\perp = DT_1 = \frac{1}{2}DT_1 = (1/\sqrt{5})DI_\perp$, from which $h = D/\sqrt{5}$. With this geometry, the lamination wall area per unit area of specimen is $[2T - 4(D/\sqrt{5})]/3D$, so the $D$-dependent part of the lamination wall energy per unit area of specimen is $E'_w = 2\gamma T / 3D$. The anisotropy energy of the closure domains per unit area of specimen is $E_K = K_1 D / (7\sqrt{5})$. Hence $E'$, the $D$-dependent part of the total energy is $E'_w + E_K$. Minimizing $E'$ with respect to $D$ shows the equilibrium value $D_0$ to be

$$D_0 = (14/\sqrt{5})\gamma T / 3K_1)^{1/3} = 3.23(\gamma T / K_1)^{1/3}. \quad \text{(5)}$$

Introducing $K_1 = 36 \text{kJ m}^{-3}$ for Fe + 3 mass% Si (Bozorth 1951) and taking $T = 16 \mu\text{m}$, the value of $D_0$ is 1.92 $\mu\text{m}$, so that on the specimen surface the corresponding stripe period $3D_0' = 3D_0 \sec \beta = 6.3 \mu\text{m}$. This is sufficiently close to the range of observed values, 8 $\mu\text{m}$ to 14 $\mu\text{m}$, in the specimen area under discussion to justify confidence in the model structure proposed in figure 6. Factors that in reality will lead to an increase of equilibrium value $D_0$ above that derived from (5), and hence closer agreement with observation, are considered below.

6. Discussion

In the domain structure shown in figure 6 the net magnetization $\vec{I}$ is normal to the plane of the section. Its magnitude averaged over the lamination cross-section is $\frac{1}{2}(P_1 + 2P_2) = 0.745I_\perp$, and averaged over the whole specimen cross-section is $(T - h)/T$
times this value. The geometric construction for perfect flux closure does not require the domains magnetized alternately parallel to [152] and [152] to be of equal depth. Application of an external field with a component in the direction [152], say, would cause closures with $I_s$ parallel to [152] to deepen, and those with $I_s$ in the opposite direction to shrink, a process that can proceed until the latter closures are eliminated, at which point the mean depth of closure domains is doubled, with consequential doubling of closure domain volume per unit area of specimen, from $D/\sqrt{5}$ to $2D/\sqrt{5}$. If inequality of [152] and [152] magnetizations develops, it does not have to do so to similar extent at top and bottom surfaces. An elastic bend of the specimen about an axis normal to figure 6 that would put the top surface in tension would favour increase in closure domain volume at the top surface relative to that at the bottom surface. When describing the X-ray topographs shown in figure 4, the slight bend of the specimen was mentioned. The bend axis makes a fairly small angle with [201], with curvature such as to put the X-ray exit surface in a state of tension, the extension in this surface being about $0.5 \times 10^{-5}$, not negligible compared with $\lambda_{100} = 2.7 \times 10^{-5}$. It would be closure domains in the upper right (A₂C plus A₂C) field rather than in the lower left field that would come under the influence of these surface stresses, but no difference between diffraction contrast in [201] stripes compared with [021] stripes that could be directly attributed to difference in their closure domain structures is seen. A strong argument against a component of magnetization transverse to the stripes is that flux matching at the field boundary would require flux reversals in this transverse magnetization to extend from field boundary corners into the B₂C plus B₂C stripe field in a manner corresponding to the 180° walls in the lamination structure that extend from field boundary corners into the upper field of A₂C plus A₂C lamination. There is not the slightest evidence for the consequential disturbance of diffraction contrast uniformity along stripes in the lower stripe field. However, there is a way in which the perfect-closure construction allows an increase in closures volume by up to a factor of two compared with figure 6 without a resultant magnetization component transverse to the stripes. This is achieved by deepening closures magnetized parallel to, say, [152] at the top surface and equally deepening closures parallel to [152] at the bottom surface, to produce a circulating flux pattern. Geometrically permissible though this may be, such change finds no favour compared with the structure drawn in figure 6. Any increase in closure domain volume over that in figure 6 can but lead to greater difference between $D_0$ calculated from equation (5) in §5 and its value derived from the observed stripe period.

To see what is required to bring calculated and observed $D$ values closer, consider the various contributions to total energy and how they differ according to whether $D$ is 1.92 μm or 3.8 μm, the latter value being that indicated by the 12.5 μm stripe period. The magnetostrictive strain energy per unit area of specimen due to the lamination structure, $E_s$, calculated in §5 to be 0.61 mJ m$^{-2}$ when $D \ll T$, will be raised as retaining about the same value, 0.6 mJ m$^{-2}$, when $D/T = 0.12$, and slightly less, say 0.5 mJ m$^{-2}$, when $D/T = 0.24$. The energy density in the closure domains is taken to be $\frac{1}{2}Y\lambda_{100}^2 \approx 74$ J m$^{-3}$, giving energies per unit area of specimen 0.06 mJ m$^{-2}$ and 0.13 mJ m$^{-2}$ for $D = 1.92$ μm and 3.8 μm respectively. An important contribution to the total energy comes from the walls between closure domains and lamination domains. These are much closer in character to (110)-plane 90° walls (for which $\gamma = 1.4$ mJ m$^{-2}$ (Kaczér 1957)) than to the cube-plane 90° walls in the lamination structures. For the closure domain walls a mean energy of 1.2 mJ m$^{-2}$ is
adopted. Taking into account their slopes with respect to (112) (which are correctly drawn in figure 6 to satisfy the pole-free condition), their area is 2.36 times the specimen area, so that the contribution they make to the total energy is 2.83 mJ m⁻² (and is independent of D). The expression for total lamination wall area for the figure 6 model was given in §5. The lamination wall energy and anisotropy energy will be compared for both D values, that from equation (5), \( D_o \), and that from observation, \( D_{obs} \). Listing the energies discussed, stated in mJ m⁻² rounded to the nearest 0.1 mJ m⁻², and distinguishing those corresponding to \( D_{obs} \) by placing them in parentheses, produces the following figures: lamination magnetostriction 0.6 (0.5), closure magnetostriction 0.1 (0.1), closure domain walls 2.8 (2.8), lamination walls 4.0 (1.7), anisotropy 4.4 (8.8), total 11.9 (13.9). Thus reduction by a quarter in the magnitude of the volume energy of the larger closures required when D = \( D_{obs} \) would be sufficient to swing the total energy in favour of that higher D. Now the model for a pole-free specimen surface does not require the whole closure domain volume to be magnetized parallel to the specimen surface. If the angle its magnetization makes with [010] is reduced to half or less of its surface value (24°), the anisotropy energy drops from \( K_i / 7 \) to less than \( K_i / 20 \), and becomes unimportant. It is suggested that such rotation occurs progressively from specimen surface inwards towards the triple junction with the lamination domains. (The relatively high values of the rotational permeability \( \mu_r = 1 + (2 \pi f / K) \approx 55 \) for Fe + 3 mass % Si facilitates such rotation (Williams et al. 1949).) As regards satisfying pole-free conditions at boundaries separating closure domains from domains in the lamination structure no difficulties would arise if the lower half of the closure domain volumes were magnetized in the direction ca. 12° off [010] towards [152] (and its inverse); the domain walls then become curved as sketched in the top right of figure 6, no significant increase in their area resulting. Against the approximate halving achieved in the volume integral of anisotropy energy in the closures must be set the energy cost of the volume distribution of free pole that continuous rotation of magnetization will entail. However, assuming equilibrium is attained when about half the anisotropy energy reduction is absorbed by the energy of the volume pole density created, there should still be sufficient reduction in the contribution of closure volume energy to total energy to displace the minimum in the latter towards occurring with D close to \( D_{obs} \) rather than to \( D_o \).

References


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